Curso :Teoria Cuantica de Campos by:Javier Garcia ejercicio cap.8, valor esperado.

Hallar (1)

$$< \phi_a\phi_b\phi_c\phi_d >= \frac{1}{Z[0]}(\frac{\partial}{\partial j_a}\frac{\partial}{\partial j_b}\frac{\partial}{\partial j_c}\frac{\partial}{\partial j_d}Z[J]\Big|_{J=0})(1)$$

$$Z[J] = \int \mathscr{D}\phi exp(-S[\phi] + \phi^T J) = exp(\frac{1}{2m^2}J^T A^{-1}J)(\frac{(\sqrt{2\pi})^n}{m^n\sqrt{detA}})(2)$$

$$Z[0] = \frac{(\sqrt{2\pi})^n}{m^n\sqrt{detA}}(3)$$

Introducimos (3) y (2) en (1)

$$< \phi_a\phi_b\phi_c\phi_d >= ((\frac{(\sqrt{2\pi})^n}{m^n\sqrt{detA}}))^{-1}(\frac{(\sqrt{2\pi})^n}{m^n\sqrt{detA}})(\frac{\partial}{\partial j_a}\frac{\partial}{\partial j_b}\frac{\partial}{\partial j_c}\frac{\partial}{\partial j_d}exp(\frac{1}{2m^2}J^T A^{-1}J)\Big|_{J=0}\quad(1)$$

$$< \phi_a\phi_b\phi_c\phi_d >= (\frac{\partial}{\partial j_a}\frac{\partial}{\partial j_b}\frac{\partial}{\partial j_c}\frac{\partial}{\partial j_d}exp(\frac{1}{2m^2}J^T A^{-1}J)\Big|_{J=0})(1)$$

Voy a utilizar la notacion de indices con el convenio de suma de einstein, pero haciendo referencia solo al sentido 'combinatorio' sin tener en cuenta bases ni duales, ni nada por el estilo.

Hare todos los calculos en bruto y al final eliminare aquellas expresiones que se anulen y dire el porque.

$$J^T A^{-1}J = J_i B^{ik} J_k(4)$$

$$a = \frac{1}{2m^2}(5)$$

Combinando (1),(4),(5)

$$< \phi_a\phi_b\phi_c\phi_d >= (\frac{\partial}{\partial j_a}\frac{\partial}{\partial j_b}\frac{\partial}{\partial j_c}\frac{\partial}{\partial j_d}exp(aJ_i B^{ik} J_k)\Big|_{J=0})(1)$$

Compactificamos mas la notacion, para no liarnos.

$$\Lambda = exp(aJ_i B^{ik} J_k)(6)$$

$$aJ_i B^{ik} J_k = \Omega(7)$$

$$\frac{\partial}{\partial j_a} = \partial_1, \frac{\partial}{\partial j_b} = \partial_2, \frac{\partial}{\partial j_c} = \partial_3, \frac{\partial}{\partial j_d} = \partial_4(8)$$

Combinando (1),(4),(5),(6),(8)

$$< \phi_a\phi_b\phi_c\phi_d >= \partial_1\partial_2\partial_3\partial_4\Lambda\Big|_{J=0}\quad(1)$$

Mas tarde usaremos (9)

$$\partial_p(\Omega) = \partial_p(aJ_iB^{ik}J_k) = a(\partial_p J_iB^{ik}J_k\delta_p^i + \partial_p J_iB^{ik}J_k\delta_p^k) = a(B^{pk}J_k + J_iB^{ip})(9)$$

Ahora comenzamos los calculos en (1), aplicamos ∂_4

$$\partial_4\Lambda = \partial_4 exp(aJ_iB^{ik}J_k) = \Lambda[\partial_1(\Omega)](10)$$

(10) en (1)

$$< \phi_a\phi_b\phi_c\phi_d >= \partial_1\partial_2\partial_3\Bigg(\Lambda[\partial_4(\Omega)]\Bigg)\Bigg|_{J=0}(1)$$

Aplicamos ∂_3

$$\partial_3\Lambda[\partial_4(\Omega)] = (\partial_3\Lambda)[\partial_4(\Omega)] + (\Lambda)[\partial_3\partial_4(\Omega)] = (\Lambda)\big([\partial_3(\Omega)][\partial_4(\Omega)] + [\partial_3\partial_4(\Omega)]\big)(11)$$

(11) en (1)

$$< \phi_a\phi_b\phi_c\phi_d >= \partial_1\partial_2\Bigg((\Lambda)\big([\partial_3(\Omega)][\partial_4(\Omega)] + [\partial_3\partial_4(\Omega)]\big)\Bigg)\Bigg|_{J=0}(1)$$

A partir de aqui y hasta el final, obviare que hay que evaluar (1)de (J) en (0).
Aplicamos ∂_2

$$\partial_2\Big((\Lambda)\big([\partial_3(\Omega)][\partial_4(\Omega)] + [\partial_3\partial_4(\Omega)]\big)\Big) = (\partial_2\Lambda)\big([\partial_3(\Omega)][\partial_4(\Omega)] + [\partial_3\partial_4(\Omega)]\big) + (\Lambda)\partial_2\big([\partial_3(\Omega)][\partial_4(\Omega)] + [\partial_3\partial_4(\Omega)]\big) =$$

$$= (\Lambda)\big([\partial_2(\Omega)][\partial_3(\Omega)][\partial_4(\Omega)] + [\partial_2(\Omega)][\partial_3\partial_4(\Omega)]\big) + (\Lambda)\big([\partial_2\partial_3(\Omega)][\partial_4(\Omega)] + [\partial_3(\Omega)][\partial_2\partial_4(\Omega)] + [\partial_2\partial_3\partial_4(\Omega)]\big) =$$

$$= (\Lambda)\Bigg(\big([\partial_2(\Omega)][\partial_3(\Omega)][\partial_4(\Omega)] + [\partial_2(\Omega)][\partial_3\partial_4(\Omega)]\big) + \big([\partial_2\partial_3(\Omega)][\partial_4(\Omega)] + [\partial_3(\Omega)][\partial_2\partial_4(\Omega)] + [\partial_2\partial_3\partial_4(\Omega)]\big)\Bigg)(12)$$

(12) en (1)

$$< \phi_a\phi_b\phi_c\phi_d >= \partial_1\Bigg((\Lambda)\Bigg(\big([\partial_2(\Omega)][\partial_3(\Omega)][\partial_4(\Omega)] + [\partial_2(\Omega)][\partial_3\partial_4(\Omega)]\big)+$$

$$+\big([\partial_2\partial_3(\Omega)][\partial_4(\Omega)] + [\partial_3(\Omega)][\partial_2\partial_4(\Omega)] + [\partial_2\partial_3\partial_4(\Omega)]\big)\Bigg)\Bigg)(1)$$

Por ultimo aplicamos ∂_1

$$(\Lambda)\Bigg(\big([\partial_1(\Omega)][\partial_2(\Omega)][\partial_3(\Omega)][\partial_4(\Omega)] + [\partial_1(\Omega)][\partial_2(\Omega)][\partial_3\partial_4(\Omega)]\big) + \big([\partial_1(\Omega)][\partial_2\partial_3(\Omega)][\partial_4(\Omega)]$$

$$+[\partial_1(\Omega)][\partial_3(\Omega)][\partial_2\partial_4(\Omega)] + [\partial_1(\Omega)][\partial_2\partial_3\partial_4(\Omega)]\big)\Bigg) + (\Lambda)\Bigg(\big([\partial_1\partial_2(\Omega)][\partial_3(\Omega)][\partial_4(\Omega)]+$$

$$+[\partial_2(\Omega)]([\partial_1\partial_3(\Omega)][\partial_4(\Omega)] + [\partial_3(\Omega)][\partial_1\partial_4(\Omega)]) + [\partial_1\partial_2(\Omega)][\partial_3\partial_4(\Omega)] + [\partial_2(\Omega)][\partial_1\partial_3\partial_4(\Omega)]\big)+$$

$$+\big([\partial_1\partial_2\partial_3(\Omega)][\partial_4(\Omega)] + [\partial_2\partial_3(\Omega)][\partial_1\partial_4(\Omega)] + [\partial_1\partial_3(\Omega)][\partial_2\partial_4(\Omega)] + [\partial_3(\Omega)][\partial_1\partial_2\partial_4(\Omega)] + [\partial_1\partial_2\partial_3\partial_4(\Omega)]\big)\Bigg)(1)$$

Tengamos en cuenta la ecuacion(9)

$$\partial_p(\Omega) = \partial_p(aJ_iB^{ik}J_k) = a(\partial_pJ_iB^{ik}J_k\delta^i_p + \partial_pJ_iB^{ik}J_k\delta^k_p) = a(B^{pk}J_k + J_iB^{ip})(9)$$

Si iteramos el proceso

$$\partial_q\partial_p(\Omega) = \partial_q\partial_p(aJ_iB^{ik}J_k) = a\partial_q(\partial_pJ_iB^{ik}J_k\delta^i_p + \partial_pJ_iB^{ik}J_k\delta^k_p) = a\partial_q(B^{pk}J_k + J_iB^{ip}) = a(\partial_qB^{pk}J_k\delta^k_q + \partial_qJ_iB^{ip}\delta^i_q) =$$

$$= a(B^{pq} + B^{qp})(13)$$

Teniendo en cuenta la simetria de A^{-1}

$$\partial_q\partial_p(\Omega) = 2a(B^{pq})(13)$$

Si volviesemos a iterar el proceso

$$\partial_l\partial_q\partial_p(\Omega) = \partial_l 2a(B^{pq}) = 0$$

Y por lo tanto todos aquellos terminos que contengan mas de dos derivadas parciales cruzadas seran eliminados.
*Solo sobreviran terminos en parejas de derivadas parciales.

$$(\Lambda)\bigg(\big([\partial_1(\Omega)][\partial_2(\Omega)][\partial_3(\Omega)][\partial_4(\Omega)] + [\partial_1(\Omega)][\partial_2(\Omega)][\partial_3\partial_4(\Omega)]\big) + \big([\partial_1(\Omega)][\partial_2\partial_3(\Omega)][\partial_4(\Omega)]$$

$$+[\partial_1(\Omega)][\partial_3(\Omega)][\partial_2\partial_4(\Omega)]\big)\bigg) + (\Lambda)\bigg(\big([\partial_1\partial_2(\Omega)][\partial_3(\Omega)][\partial_4(\Omega)]+$$

$$+[\partial_2(\Omega)]([\partial_1\partial_3(\Omega)][\partial_4(\Omega)] + [\partial_3(\Omega)][\partial_1\partial_4(\Omega)]) + [\partial_1\partial_2(\Omega)][\partial_3\partial_4(\Omega)]+$$

$$+[\partial_2\partial_3(\Omega)][\partial_1\partial_4(\Omega)] + [\partial_1\partial_3(\Omega)][\partial_2\partial_4(\Omega)]\big)\bigg)(1)$$

*Ademas, tenemos que evaluar las (J) en [0] y por tanto, aquellos terminos que contengan

$$[\partial_p(\Omega)] = a(B^{pk}J_k + J_iB^{ip})$$

Seran iguales a cero y aqui, cero por algo es cero, ademas, $\Lambda\bigg|_{J=0} = 1$por tanto.

$$< \phi_a\phi_b\phi_c\phi_d >= \bigg([\partial_1\partial_2(\Omega)][\partial_3\partial_4(\Omega)] + [\partial_2\partial_3(\Omega)][\partial_1\partial_4(\Omega)] + [\partial_1\partial_3(\Omega)][\partial_2\partial_4(\Omega)]\bigg)(1)$$

combinando (1), (13) , (4) ,(5) y (9).

$$< \phi_a\phi_b\phi_c\phi_d >= 2a(A^{-1}_{12})2a(A^{-1}_{34}) + 2a(A^{-1}_{23})2a(A^{-1}_{14}) + 2a(A^{-1}_{13})2a(A^{-1}_{24})$$

$$< \phi_a\phi_b\phi_c\phi_d >= 4a^2(A^{-1}_{ab})(A^{-1}_{cd}) + 4a^2(A^{-1}_{bc})(A^{-1}_{ad}) + 4a^2(A^{-1}_{ac})(A^{-1}_{bd})$$

$$< \phi_a\phi_b\phi_c\phi_d >= \frac{1}{m^4}\Big(A^{-1}_{ab}A^{-1}_{cd} + A^{-1}_{bc}A^{-1}_{ad} + A^{-1}_{ac}A^{-1}_{bd}\Big)(1)$$